**P.7**

**Solving Inequalities Algebraically and Graphically**

## Solving Absolute Value Inequalities

Let u be an algebraic expression in x and let *a* be a real number with *a* $\geq 0$.

1. If $|u| < a$, then *u* is in the interval (-*a*, *a*). That is, $|u| < a$ if and only if -*a* < *u* < *a*.
2. If $|u| > a$, then u is in the interval $(-\infty, -a)$ or $(a, \infty)$, that is $|u| > a$ if and only if *u* < -*a* or *u* > *a*.

   (The inequalities can be replaced with $\leq$ $\geq$)

Ex.

Solve $|4-3x|-2<4$.

$$|4-3x|-2<4$$

$$|4-3x|<6$$

$$-6<4-3x<6$$

$$-10<-3x<2$$

$$\frac{10}{3}>x>\frac{-2}{3} \quad \text{or} \quad \left(\frac{-2}{3},\frac{10}{3}\right)$$

## Solving Quadratic Inequalities

Ex. Solve $2x^2+7x>15$.

First we will subtract 15 from both sides of the inequality to obtain $2x^2+7x-15>0$.

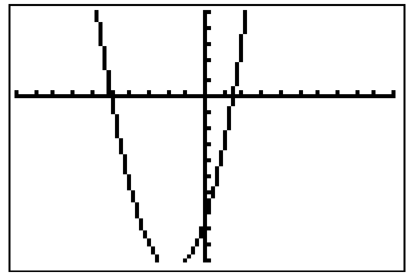
Next we will solve the corresponding quadratic equation $2x^2+7x-15=0$.

$$2x^2+7x-15=0$$

$(2x-3)(x+5)=0$ so, $2x-3=0$ *or* $x+5=0$

Which gives $x = \frac{3}{2} \quad or \quad x = -5$

Looking at the graph of $2x^2 + 7x - 15 = 0$



we can see that the points on the graph that

are above the y-axis are positive. This happens

when x < -5 and x > 3/2. In interval notation we

have $(-\infty, -5) \cup \left(\frac{3}{2}, \infty\right)$.

**<u>Projectile Motion</u>**

Suppose an object is launched vertically from a point $s_0$ feet above the ground with an initial velocity of $v_0$ feet per second. The vertical position *s* (in feet) of the object *t* seconds after it is launched is: $s = -16t^2 + v_0 t + s_0$.

Ex. A projectile is launched straight up from ground level with an initial velocity of 272 ft/sec.

a) When will the projectile's height above ground be 960 feet?

b) When will the projectile's height above ground be more than 960 feet?

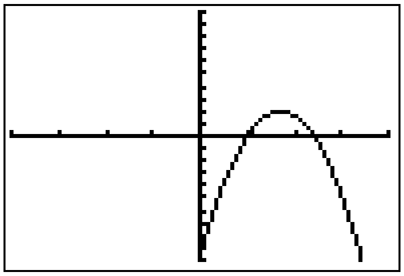
c) When will the projectile's height above ground be less than or equal to 960 feet?

Solution: Here $s_0$ = 0 and $v_0$ = 272. So, the projectiles height is $s = -16t^2 + 272t$.

a) We need to determine when s = 960.

$$\begin{aligned}
&960 = -16t^2 + 272t \\
&-16t^2 + 272t - 960 = 0 \\
&-16(t^2 - 17t + 60) = 0 \\
&-16(t-5)(t-12) = 0 \\
&t = 5 \quad or \quad t = 12
\end{aligned}$$

The projectile is 960 ft above ground twice. At t = 5 seconds on the way up, and t = 12 seconds on the way down.

b) 

Looking at the graph we can see that between 5 seconds and 12 seconds the projectile is at least 960 ft above ground.

c) The projectile is less than 960 ft above ground between 0 and 5 seconds and after 12 seconds.